

bule of Air as *B* has been above a larger as *A*, which Globule has not risen upward to *C*, and so to *D*, but been thrust downwards to *A*, whence it was distant about two hair's breadths, and immediately upon touching united therewith. I have likewise observed, that a little Air-bubble as *G*, loosening itself from the straw, when a larger Bubble, as *F*. was underneath it, has there rested immovable in the Liquor, when at the same time other much smaller Bubbles have risen to the top thereof. The reason of the standing still of the Bubble *G*, I suppose was from a double motion it is impelled to, the one upwards from its being specifically higher than the Liquor, the other downwards, by which it was protruded, to joyn with the other larger Bubble *F*. Tho' I have seen several Effects of Sympathy, if we may so call it, yet I never saw any so manifest as this, of the descending of a Bubble contrary to its levity, to unite with another.

IV. *An Instance of the Excellence of the Modern ALGEBRA, in the Resolution of the Problem of finding the Foci of Optick Glasses universally.* By E. Halley, S. R. S.

THE Excellence of the *Modern Geometry* is in nothing more evident, than in those full and adequate Solutions it gives to Problems; representing all the possible Cases at one view, and in one general Theorem many times comprehending whole Sciences; which deduced at length into Propositions, and demonstrated after the manner of the *Ancients*, might well become the Subjects of large Treatises: For whatsoever Theorem solves the most complicated Problem of the kind, does with a due Reduction reach all the subordinate Cases. Of this

I

I now design to give a notable Instance in the Doctrine of *Dioptricks*.

This Dioptrick Problem is that of finding the *Focus* of any sort of *Lens*, exposed either to Converging, Diverging or parallel Rays of Light, proceeding from, or tending to a given Point in the *Axis* of the *Lens*, be the *ratio* of *Refraction* what it will, according to the nature of the Transparent Material whereof the *Lens* is formed, and also with allowance for the thickness of the *Lens* between the *Vertices* of the two Spherical Segments. This Problem being solved in one Case, *mutatis mutandis* will exhibit Theorems for all the possible Cases, whether the *Lens* be *Double-Convex* or *Double-Concave*, *Plano-Convex* or *Plano-Concave*, or *Convexo-Concave*, which sort are usually called *Menisci*. But this is only to be understood of those Beams which are nearest to the *Axis* of the *Lens*, so as to occasion no sensible difference by their Inclination thereto; and the *Focus* here formed is by *Dioptrick Writers* commonly called the principal *Focus*, being that of use in *Telescopes* and *Microscopes*.

Let then (in Fig. 18.) $BE\beta$ be a double Convex *Lens*, C the Center of the Segment EB, and K the Center of the Segment $E\beta$, $B\beta$ the thickness of the *Lens*, D a point in the *Axis* of the *Lens*; and it is required to find the point F, at which the Beams proceeding from the point D, are collected therein, the *ratio* of *Refraction* being as m to n . Let the distance of the object $DB = DA = d$ (the point A being supposed the same with B, but taken at a distance therefrom, to prevent the coincidence of so many Lines) the *Radius* of the Segment towards the Object CB or CA = r , and the *Radius* of the Segment from the Object K β or K $\alpha = \rho$, and let $B\beta$ the thickness of the *Lens* be = t , and then let the Sine of the Angle of incidence DAG be to the sine of the refracted Angle HAG or CA ϕ as m to n ; And in very small Angles the Angles them-

themselves will be in the same proportion; whence it will follow that,

As d to r , so the Angle at C to the Angle at D, and $d+r$ will be as the Angle of incidence GAD; and again as m to n , so $d+r$ to $\frac{dn+r^2n}{m}$ which will be as the Angle GAH = CA ϕ . This being taken from ACD which is as d , will leave $\frac{m-nd-nr}{m}$ analogous to the Angle A ϕ D, and the sides being in this case proportional to the Angles they subtend, it will follow, that as the Angle A ϕ D is to the Angle AD ϕ , so is the side AD or BD to A ϕ or B ϕ : that is B ϕ will be $\frac{m d r}{m-nd-nr}$ which shews in what point the beams proceeding from D would be collected by means of the first Refraction; but if nr cannot be subtracted from $m-nd$, it follows that the Beams after Refraction do still pass on diverging, and the point ϕ is on the same side of the Lens beyond D. But if nr be equal to $m-nd$ then they proceed parallel to the Axis, and the point ϕ is infinitely distant.

The point ϕ being found as before, and B ϕ =B β being given, which we will call δ , it follows by a process like the former, that βF or the focal distance sought,

is equal to $\frac{\delta p n}{m-n\delta+m\beta} = f$. And in the room of δ sub-

stituting B ϕ =B $\beta = \frac{m d r}{m-nd-nr} - t$, putting p for $\frac{n}{m-n}$ after due reduction this following Equation will arise,

$$\frac{m p d r p - n d p t + n p r p t}{m d r + m d e - m p r e - m - n d t + n r t} = f.$$

Which Theorem, however it may seem operose, is

is not so, considering the great number of *data* that enter the Question, and that one half of the terms arise from our taking in the thickness of the *Lens*, which in most cases can produce no great effect, however it was necessary to consider it, to make our Rule perfect. If therefore the *Lens* consist of *Glass*, whose Refraction is

as 3 to 2 'twill be
$$\frac{6 dr \rho - 2 d \rho t + 4 r \rho t}{3 dr + 3 d \rho - 6 r \rho - d t + 2 r t} = f.$$

If of *Water*, whose Refraction is as 4 to 3 the Theorem will stand thus
$$\frac{12 dr \rho - 3 d \rho t + 9 r \rho t}{4 dr + 4 d \rho - 12 r \rho - d t + 3 r t} = f.$$

If it could be made of *Diamant*, whose Refraction is as 5 to 2, it would be
$$\frac{10 dr \rho - 2 d \rho t + 5 r \rho t}{5 dr + 5 d \rho - 10 r \rho - d t + 2 r t} = f.$$

And this is the universal Rule for the *foci* of double Convex Glasses expos'd to Diverging Rays. But if the thickness of the *Lens* be rejected as not sensible, the Rule will be much shorter, *viz.*

$$\frac{\rho dr \rho}{dr + d \rho - r \rho} = f,$$
 or in *Glass*
$$\frac{2 dr \rho}{dr + d \rho - 2 r \rho} = f.$$

all the terms wherein *t* is found being omitted, as equal to nothing. In this case, if *d* be so small, as that $2 r \rho$ exceed $dr + d \rho$, then will it be $-f$, or the *focus* will be Negative, which shews that the Beams after both Refractions still proceed Diverging.

To bring this to the other Cases, as of Converging Beams, or of Concave Glasses, the Rule is ever compos'd of the same terms, only changing the signs of $+$ and $-$; for the distance of the point of Concourse of converging Beams, from the point B, or the first surface of the *Lens*, I call a negative distance or $-d$; and the Radius of a Concave *Lens* I call a negative Radius or $-r$ if it be the first surface, and $-\rho$, if it be the second surface. Let then converging Beams fall on a double Con-

vex of Glafs, and the Theorem will stand thus

$$\frac{-2 dr \rho}{-dr - d\rho - 2r\rho} = +f.$$
 which shews that in this case the *Focus* is always affirmative.

If the *Lens* were a *Meniscus* of Glafs, exposed to diverging Beams, the Rule is

$$\frac{-2 dr \rho}{-dr + d\rho + 2r\rho} = f.$$
 which is affirmative when $2r\rho$ is less than $dr - d\rho$, otherwise negative : But in the case of converging Beams falling on the same *Meniscus*, 'twill be

$$\frac{+2 dr \rho}{+dr - d\rho + 2r\rho} = f.$$

and it will be $+f$ whilst $d\rho - dr$ is less than $2r\rho$, but if it be greater than $2r\rho$, it will always be found negative or $-f$. If the *Lens* be double Concave, the *focus* of converging Beams is negative, where it was affirmative in the case of diverging Beams on a double Convex, viz.

$$\frac{-2 dr \rho}{+dr + d\rho - 2r\rho} = f,$$
 which is affirmative only when $2r\rho$ exceeds $dr + d\rho$: But diverging Beams passing a double Concave have always a negative *focus*,

viz.
$$\frac{-2 dr \rho}{+dr + d\rho + 2r\rho} = -f.$$

The Theorems for Converging Beams are principally of use to determine the *focus* resulting from any sort of *Lens* placed in a Telescope, between the *focus* of the Object-glass and the Glafs it self ; the distance between the said *focus* of the Object-glass and the interposed *Lens* being made $= -d$.

I here suppose my Reader acquainted with the Rules of Analytical Multiplication and Division, as that $+$ multiplied by $+$ makes the product $+$, $+$ by $-$ makes $-$, and $-$ by $-$ makes $+$, so dividing $+$ by $+$ makes the Quote $+$, $+$ by $-$ makes $-$, and $-$ by $-$ makes $+$, which will be necessary to be understood in the preceding Examples.

In case the Beams are parallel, as coming from an infinite distance, (which is supposed in the case of Telescopes) then will d be supposed infinite, and in the

Theorem $\frac{p d \rho r}{d r \times d \rho - p r \rho}$ the Term $p r \rho$ vanishes, as being finite, which is no part of the other infinite terms and dividing the remainder by the infinite part d , the

Theorem will stand thus $\frac{p \rho r}{r \times \rho} = f$, or in Glafs,

$$\frac{2 r \rho}{r \times \rho} = f.$$

In case the *Lens* were *Plano-Convex* exposed to diverging Beams, instead of $\frac{p d \rho r}{d r \times d \rho - p r \rho}$, r being infinite,

it will be $\frac{p d \rho}{d - p \rho} = f$. or $\frac{2 d \rho}{d - 2 \rho} = f$, if the *Lens* be Glafs.

If the *Lens* be *Double-Convex*, and r be equal to ρ , as being formed of Segments of equal Spheres, then will

$\frac{p d e r}{d r \times d \rho - p r \rho}$ be reduced to $\frac{p d r}{2 d - p r} = f$; and

in case d be infinite, then it will yet be farther contracted to $\frac{1}{2} p r$, and p being $= \frac{n}{m - n}$ the focal distance in Glafs will be $= r$, in Water $1 \frac{1}{2} r$, but in Diamant $\frac{3}{2} r$.

I am sensible that these Examples are too much for the compleat Analyst, though I fear too little for the less Skillful, it being very hard, if possible, in such matters, so to write as to give satisfaction to both; or to please the one, and instruct the other. But this may suffice to shew the extent of our Theorem, and how easy a Reduction adapts any one case to all the rest.

Nor is this only useful to discover the *focus* from the other proposed *data*, but from the *focus* given, we may thereby determine the distance of the Object, or from the *focus* and distance given, we may find of what Sphere it is requisite to take another Segment, to make any given Segment of another Sphere cast the beams from the distance d to the *focus* f . As likewise from the *Lens*, *focus*, and distance given, to find the *ratio* of Refraction, or of m to n , requisite to answer those *data*. All which it is obvious, are fully determined from the Equation we have hitherto used, *viz.* $p d e r = d r f \mp d e f - p r e f$, for to find d the Theorem is

$$\frac{p r e f}{r f \mp e f - p e r} = d, \text{ the distance of the Object.}$$

For e the Rule is
$$\frac{d r f}{p d r \mp d f \mp p r f} = e$$

But for p will be
$$\frac{d r f \mp d e f}{d e r \mp f e r} = p,$$
 which latter determines the *ratio* of Refraction, m being to n as $1 \mp p$ to p .

I shall not expatiate on these Particulars, but leave them for the exercise of those that are desirous to be informed in Optical Matters, which I am bold to say are comprehended in these three Rule, as fully as the most inquisitive can desire them, and in all possible cases; regard being had to the Signs \mp and $-$, as in the former cases of finding the *focus*. I shall only shew two considerable uses of them; the one to find the distance whereat an Object, being placed shall by a given *Lens* be represented in a *Species* as large as the Object it self, which may be of singular use, in drawing Faces, and other things in their true Magnitude, by transmitting the *Species* by a *Glass* into a dark Room, which will not only give the true Figure and Shades, but even the Colours themselves, almost as vivid as the Life. In this case

case d is equal to f , and substituting d for f in the Equation, we shall have $p dr \xi = d dr + dd\xi - d p \xi r$, and dividing all by d . $p r \xi = dr + d\xi - p r \xi$; that is $\frac{2 p r \xi}{r + \xi} = d$; but if the two Convexities be of the same

Sphere so as $r = \xi$ then will the distance be $= p r$, that is, if the *Lens* be *Glass* $= 2 r$, so that if an Object be placed at the Diameter of the Sphere distant, in this case the *focus* will be as far within as the Object is without, and the *Species* represented thereby will be as big as the Life; but if it were a *Plano-Convex*, the same distance will be $= 2 p r$, or in *Glass* to four times the *Radius* of the Convexity; but of this method I may perhaps entertain the Curious in some other Transaction, and shew how to magnifie or diminish an Object in any proportion assigned, (which yet will be obvious enough from what is here delivered) as likewise how to erect the Object which in this method is represented inverted.

A second use is to find what Convexity or Concavity is required, to make a vastly distant Object be represented at a given *focus*, after the one surface of the *Lens* is formed; which is but a Corollary of our Theorem for finding ξ , having p , d , r and f given; for d being infinite, that Rule becomes $\frac{r f}{p r - f} = \xi$, that is in *Glass*

$\frac{r f}{2 r - f} = p$, whence if f be greater than $2 r$, ξ becomes Negative, and $\frac{r f}{f - 2 r}$ is the *Radius* of the Concave sought.

Those that are wholly to begin with this Dioptrical Science cannot do better than to read with Attention a late Treatise of Dioptricks, published By *W. Molineux*, Esq; R. S. S. who has at large shewn the Nature of

Optick Glasses, and the Construction and Use of Microscopes and Telescopes; and though some nicely Critical have endeavoured to spy faults, and to traduce the Book, yet having long since examined it with care, I affirm, that if I can judge, it hath but two things that with any Colour may be call'd faults; the one an over-careful acknowledgment of every Trifle the Author had received from other; and the other, that he labours to make easie this curious Subject, so little understood by most, in a manner perhaps too familiar for the *Learned Critick*, and which demonstrates that it was writ *cum animo docendi*, both which require but very little Friendship or good Nature in the Reader, to pass for Vertues in an Author.

But to return to our first Theorem, which accounting for the thickness of the *Lens*, we will here again resume, *viz.*

$$\frac{m p d r p - n d p t + n p r p t}{m d r \cancel{r} m d e - m p r e - m - n d t + n r t} = f.$$

And let it be required to find the *focus* where a whole Sphere will collect the Beams proceeding from an Object at the distance d ; Here t is equal to $2r$ and $r = e$. And after due Reduction the Theorem will stand thus,

$$\frac{m p d r - 2 n d r \cancel{r} + 2 n p r r}{2 n d \cancel{r} + 2 n r - m p r} = f, \text{ but if } d \text{ be infi-}$$

nite, it is contracted to $\frac{m p r}{2 n} - r = \frac{2 n - m}{2 m - 2 n} r = f,$

wherefore a Sphere of Glas collects the Suns Beams at half the Semi-diameter of the Sphere without it, and a Sphere of Water at a whole Semi-diameter. But if the *ratio* of Refraction m to n be as 2 to 1, the *focus* falls on the opposite surface of the Sphere, but if it be of greater inequality it falls within.

Another Example shall be when a Hemisphere is exposed to parallel Rays, that is d and e being infinite, and $t = r$, and after due Reduction the Theorem results $\frac{r}{m m - m n} r = f$. That is, in Glass it is at $\frac{3}{2} r$, in Water at $\frac{5}{4} r$, but if the Hemisphere were Diamant, it would collect the Beams at $\frac{1}{3}$ of the Radius beyond the Center.

Lastly, As to the effect of turning the two sides of a *Lens* towards an Object; it is evident, that if the thickness of the *Lens* be very small, so as that you neglect it, or account $t = 0$, then in all cases the *focus* of the same *Lens*, to whatsoever Beams, will be the same, without any difference upon the turning the *Lens*: But if you are so Curious as to consider the thickness, (which is seldom worth accounting for) in the case of parallel Rays falling on a *Plano-Convex* of Glass, if the plain side be towards the Object, t does occasion no difference, but the focal distance $f = 2 r$. But when the Convex side is towards the Object, it is contracted to $2 r - \frac{2}{3} t$, so that the *focus* is nearer by $\frac{2}{3} t$. If the *Lens* be double Convex the difference is less; if a *Meniscus* greater. If the Convexity on both sides be equal, the focal length is about $\frac{1}{2} t$ shorter than when $t = 0$. In a *Meniscus* the Concave side towards the Object encreases the focal length, but the Convex towards the Object diminishes it. A General Rule for the difference arising on turning the *Lens*, where the *Focus* is Affirmative, is this

$\frac{2 r t - 2 e t}{3 r \mp 3 e - t}$, for double Convexes of differing Spheres. But for *Menisci* the same difference becomes $\frac{2 r t \mp 2 e t}{3 r - 3 e \mp t}$; of which I need give no other demon-

stration, but that by a due Reduction it will so follow from what is premised, as will the Theorems for all-sorts of Problems relating to the *foci* of Optick Glasses.

Fig: 4.

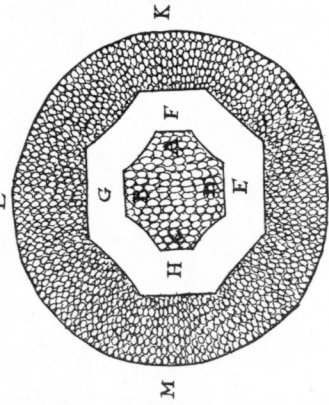


Fig: 8.

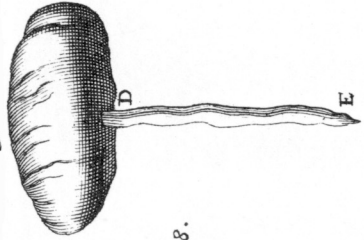


Fig: 8.

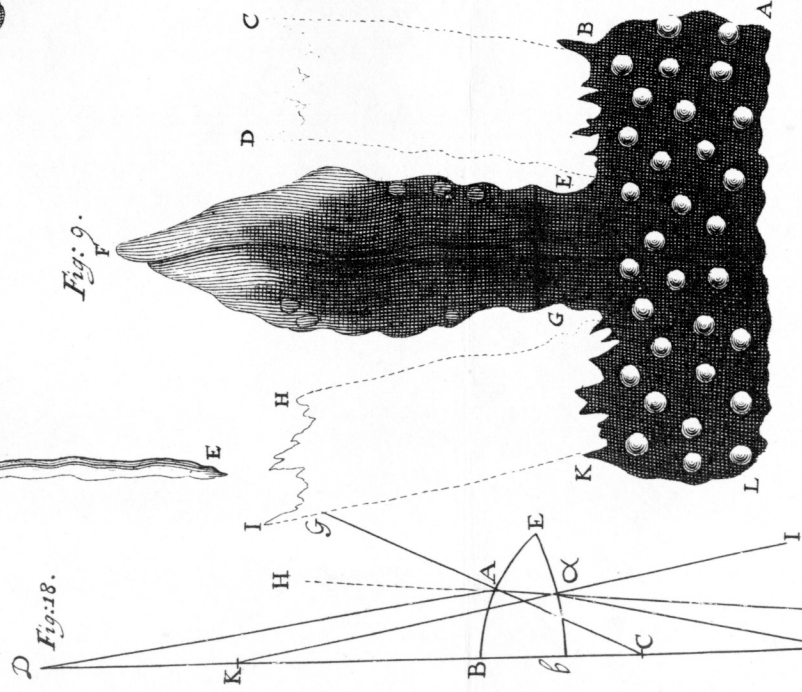


Fig: 9.

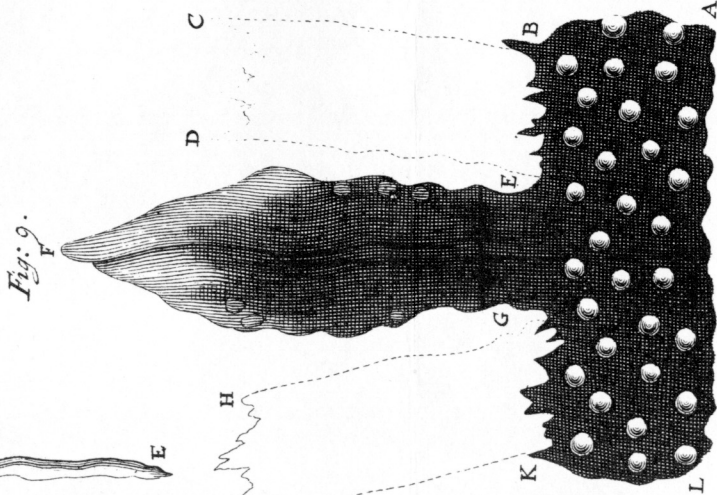


Fig: 10.

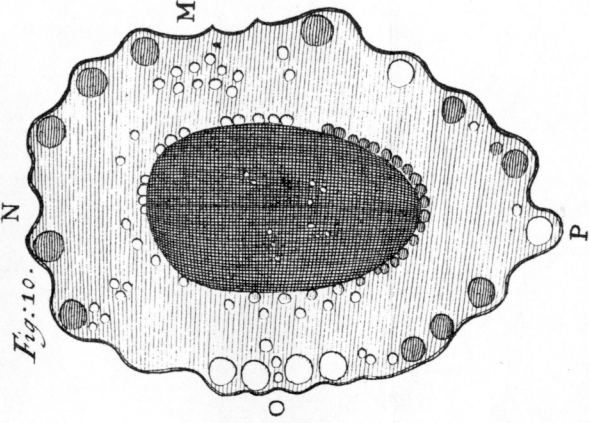


Fig: B.

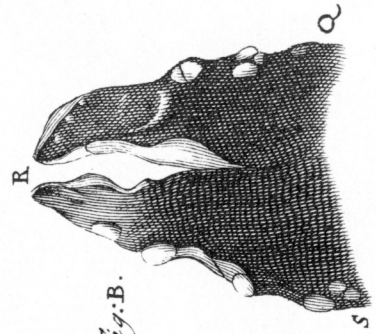


Fig: 1.

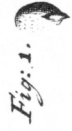


Fig: 2.

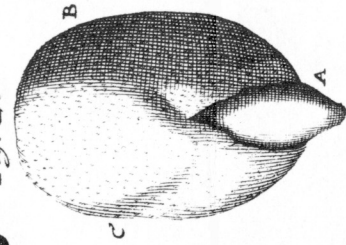


Fig: 5.

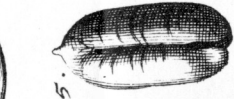


Fig: 6.

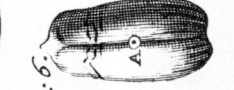


Fig: 7.

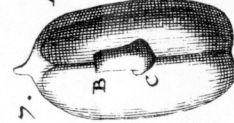


Fig: 3.

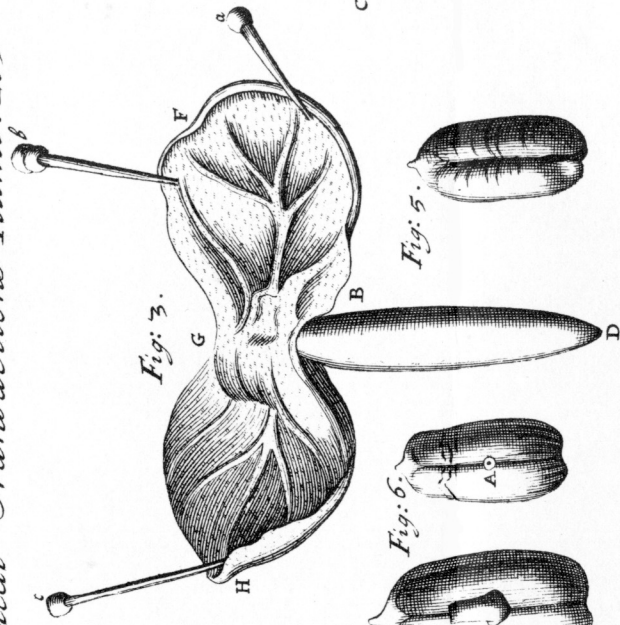


Fig: 12.

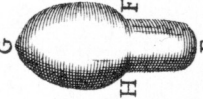


Fig: n. C.

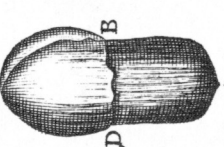


Fig: 14.

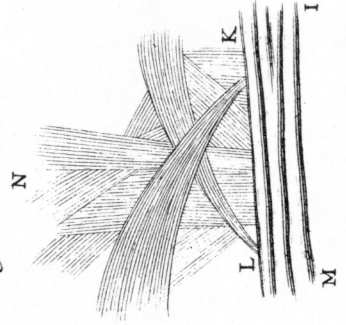


Fig: 17.

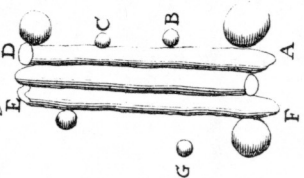
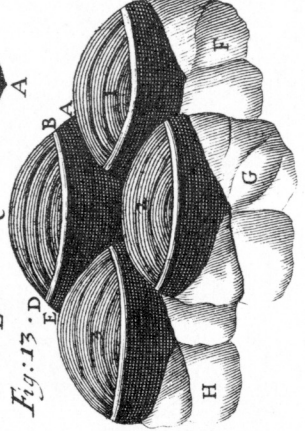
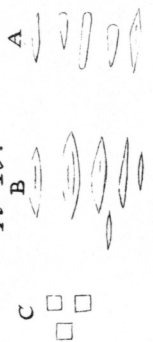


Fig: 13.



N° 16.



N° 15.

